**Indian Statistical Institute, Bangalore Centre**

M.Math. I Year, Second Semester

Semestral Examination

Functional Analysis (Back Paper)

Time: 3 hours | Date: | Instructor: T.S.S.R.K.Rao

Maximum Marks 100

Answer all the questions. You can state correctly and use any result proved in class. How ever if an answer is an immediate consequence of a result quoted, that result also need to be proved. Each question is worth 10 points.

1. Let $X$ be a normed linear space. Show that $X^*$ is a Banach space.
2. State the Open mapping theorem. State and prove the Closed graph theorem.
3. Let $\{f_n\}_{n\geq 1} \subset L^4([0,1])$ be such that $\|f_n\| \to 0$. Show that for any $g \in L^{\frac{4}{3}}([0,1])$, $\int f_n g dx \to 0$.
4. Let $A$ be a commutative Banach algebra with identity $e$. Let $I$ be a proper closed ideal. Show that the quotient space $A/I$ is a Banach algebra.
5. Show that any finite dimensional subspace of a normed linear space is closed.
6. State and prove the Banach-Alaoglu theorem.
7. Show that any separable Hilbert space is isometric to $\ell^2$.
8. Let $A$ be a Banach algebra with identity $e$. Show that for any complex homomorphim $\phi : A \to \mathbb{C}$, $ker\phi$ is a closed ideal.
9. Show that any unitary operator on a complex Hilbert space is an isometry and preserves the inner product.
10. Let $\Delta = \{z : |z| \leq 1\}$. Let $A = \{f \in C(\Delta) : f$ is analytic in the interior$\}$. Show that $A$ is a Banach algebra with identity.